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Structure analysis of the virtual Compton scattering amplitude at low energies *

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Abstract

We analyze virtual Compton scattering off the nucleon at low energies in a covariant, model-independent formalism. We define a set of invariant functions which, once the irregular nucleon pole terms have been subtracted in a gauge-invariant fashion, is free of poles and kinematical zeros. The covariant treatment naturally allows one to implement the constraints due to Lorentz and gauge invariance, crossing symmetry, and the discrete symmetries. In particular, when applied to the $ep \rightarrow e'p'\gamma$ reaction, charge-conjugation symmetry in combination with nucleon crossing generates four relations among the ten originally proposed generalized polarizabilities of the nucleon.

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I. INTRODUCTION

The derivation of the structure of the general virtual Compton scattering (VCS) amplitude from the nucleon has been a problem with a long history [1–3]. The most sophisticated treatment of the general Compton process with both the initial and final state photons off shell, $\gamma^*N \rightarrow \gamma^*N$, was presented by Tarrach in Ref. [3]. We will use this work as a starting point for our investigation of the low-energy VCS amplitude.

Whether one analyzes Compton scattering within the framework of a given theoretical model or experimentally, it is in any case desirable to perform the analysis in terms of a set of amplitudes which is solely determined by the dynamics of the VCS process with the kinematics being factored out. In the case of VCS with one or even two virtual photons, it is by no means trivial to find an adequate set of amplitudes which fulfills this requirement [2,3]. In particular, as will become obvious in the following, it is a central issue to construct tensor structures and corresponding amplitudes which are free of poles or other kinematical constraints. This problem must be addressed on a model-independent level by taking into account general symmetry principles like gauge and Lorentz invariance and discrete symmetries like parity, time reversal, and charge conjugation. We will discuss an ensemble of tensor structures and amplitudes with the desired properties for the case of $\gamma^*N \rightarrow \gamma^*N$ which then can be applied to the case $\gamma^*N \rightarrow \gamma N$ with a real photon in the final state. In particular, our results for the regular part of the VCS amplitude can be expressed in terms of even fewer functions than suggested in Ref. [3].

The process $\gamma^*N \rightarrow \gamma N$ will be analyzed at the electron laboratories MAMI (Mainz), Jefferson Lab (Newport News), and MIT-Bates by means of electron scattering off a proton target, $ep \rightarrow e'p'\gamma$ [4]. In the electron scattering process the genuine VCS amplitude interferes with the electron bremsstrahlung amplitude also known as the Bethe-Heitler process, which is completely determined by quantum electrodynamics and the electromagnetic form factors of the nucleon. We will not discuss the Bethe-Heitler mechanism in this paper.

The scheduled VCS experiments have stimulated quite a few theoretical activities. Model-independent aspects of VCS have been studied in Refs. [5–9]. The various predictions for model-dependent quantities related to VCS comprise the constituent quark model [6,10], an effective Lagrangian approach [11], calculations [12] in a coupled-channel unitary model [13], field-theoretical models like the linear sigma model [14,15], and heavy baryon chiral perturbation theory (HBChPT) [16,17] as well as the Skyrme model [18]. An overview of recent work on VCS may be found in Ref. [19].

The formalism applied most frequently to VCS at small final photon energy and large momentum transfer has been developed in Ref. [6]. In that work the regular part of the VCS amplitude has been parametrized in terms of ten generalized polarizabilities of the nucleon—three in the spin-independent and seven in the spin-dependent part of the amplitude. Recently, a general proof has been given [9] that only two of the three generalized polarizabilities in the spin-independent sector are independent of each other if charge conjugation and nucleon crossing are applied. In the present work we will analyze the spin-dependent amplitudes on the grounds of a covariant treatment. The central result of our investigation will be that due to gauge invariance, Lorentz invariance, and invariance under parity, time reversal and charge conjugation in combination with nucleon crossing the regular part of the VCS amplitude can be written in terms of only six independent generalized

polarizabilities instead of ten if one performs the same kinematical approximations as in Ref. [6].

Our paper is organized as follows: In Sec. II we briefly review the formalism of Ref. [3], adapting the notation to our conventions, and simplify the results according to our needs. Part of the derivation is contained in Appendix A. In this section we will also specify the set of amplitudes we will work with. In Sec. III we investigate the number of independent generalized polarizabilities of the nucleon if one imposes the same kinematical and symmetry constraints as in Ref. [6] but in addition requires the VCS amplitude to be invariant under the simultaneous transformation of charge conjugation and nucleon crossing. Finally, we give a brief summary in Sec. IV.

II. GENERAL STRUCTURE OF THE VCS AMPLITUDE

In this section we discuss the general form of the amplitude $\mathcal{M}^{\gamma^* \gamma}$ for the VCS reaction $\gamma^* + N \rightarrow \gamma + N$. Before going into detail let us briefly explain our notation: The initial (final) photon is characterized by the four-momentum $q^\mu = (\omega, \vec{q})$ [$q'^\mu = (\omega', \vec{q}')$], and the polarization vector $\varepsilon^\mu = (\varepsilon^0, \vec{\varepsilon})$ [$\varepsilon'^\mu = (\varepsilon'^0, \vec{\varepsilon}')$]. The four-momenta of the nucleons read $p_i^\mu = (E_i, \vec{p}_i)$, $p_f^\mu = (E_f, \vec{p}_f)$. For convenience, we introduce abbreviations for the sum of the photon and the nucleon momenta,¹

$$P = p_i + p_f, \quad Q = q + q'. \quad (1)$$

The covariant result for $\mathcal{M}^{\gamma^* \gamma}$ turns out to be a powerful tool for three reasons: First of all, it can be used to investigate the number of independent observables characterizing different kinematical approximations. We study the consequences of the restriction to the lowest-order term in the real-photon energy ω' in order to determine the number of independent generalized polarizabilities. Secondly, starting from the VCS results the transition to real Compton scattering (RCS) is simple and one is able to connect observables defined in RCS with those in VCS. In particular, the relation between the third-order spin polarizabilities, as defined by Ragusa [20] for RCS, and the generalized polarizabilities of Guichon *et al.* [6] can be obtained [21]. Finally, our covariant result is appropriate to determine the general form of the VCS amplitude in any specific frame. In this paper, we only deal with the c.m. frame.

We start our analysis of the VCS amplitude considering the most general case with two virtual photons. The amplitude can be regarded as the contraction of the VCS tensor $M^{\mu\nu}$ with the polarization vectors of the photons, evaluated between the nucleon spinors in the initial and final states,

$$\mathcal{M}^{\gamma^* \gamma^*} = -ie^2 \bar{u}(p_f, S_f) \varepsilon_\mu M^{\mu\nu} \varepsilon'_\nu{}^* u(p_i, S_i). \quad (2)$$

Throughout this paper we use the conventions of Bjorken and Drell [22], where $\mathcal{M}^{\gamma^* \gamma^*}$ is the invariant matrix element of the VCS reaction. The normalization of the nucleon spinor

¹We note that the definitions in Eq. (1) differ by a factor of 2 from those used in Ref. [3] but agree with Refs. [8] and [9].

reads $\bar{u}(p, S)u(p, S) = 1$, and we adopt Heaviside-Lorentz units where the square of the elementary charge is given by $e^2/4\pi \approx 1/137$.

In order to disentangle new information from the VCS tensor, it is useful to separate from $M^{\mu\nu}$ the contribution which is irregular in the limit $q \rightarrow 0$ or $q' \rightarrow 0$. For that purpose we divide $M^{\mu\nu}$ into a pole piece $M_A^{\mu\nu}$ and a residual part $M_B^{\mu\nu}$,

$$M^{\mu\nu} = M_A^{\mu\nu} + M_B^{\mu\nu}. \quad (3)$$

In fact, such a splitting is not unique and we will follow the *convention* of Refs. [3] and [6] of evaluating the s - und u -channel pole terms using electromagnetic vertices of the form

$$\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2), \quad q = p' - p, \quad (4)$$

where F_1 and F_2 are the Dirac and Pauli form factors of the proton, respectively. The explicit result for $M_A^{\mu\nu}$ is given in Eq. (18) of Ref. [3]. As a consequence of Low's theorem [23], any calculation of pole terms involving on-shell equivalent forms of the nucleon electromagnetic current yields the same irregular contribution to the VCS matrix element (for a proof of this claim in the context of VCS, see Sec. IV B of Ref. [7]). It is advantageous to use the particular form of Eq. (4), since the resulting $M_A^{\mu\nu}$ separately satisfies all the symmetry requirements, in particular gauge invariance. Even though this terminology is not quite precise, we will adhere to the common practice of referring to the $M_A^{\mu\nu}$ evaluated with the vertices of Eq. (4) as the “Born terms.” The corresponding $M_B^{\mu\nu}$ will variously be denoted as the regular or structure-dependent or residual or non-Born contribution. For a complete discussion of the ambiguity concerning what exactly is meant by “Born terms,” the interested reader is referred to Sec. IV of Ref. [7]. In the following, we are mainly interested in the non-Born contribution to the Compton tensor, as this part *by definition* involves the generalized polarizabilities of Ref. [6] and the low-energy constants to be defined below.

Using gauge invariance,

$$q_\mu M^{\mu\nu} = q'_\nu M^{\mu\nu} = 0, \quad (5)$$

a system of independent tensors serving as a basis of $M^{\mu\nu}$ was derived by Tarrach [3]. Once $M_A^{\mu\nu}$ and $M_B^{\mu\nu}$ are chosen to be gauge invariant, we can construct both of them by use of the same basis $M^{\mu\nu}$.

Since the work of Tarrach [3] plays an important role in our further analysis, we have summarized its results in Appendix A, in particular the representation of the Compton tensor in terms of 18 basis elements $T_i^{\mu\nu}$:

$$M_B^{\mu\nu} = \sum_{i \in J} B_i(q^2, q'^2, q \cdot q', q \cdot P) T_i^{\mu\nu}, \quad J = \{1, \dots, 21\} \setminus \{5, 15, 16\}. \quad (6)$$

At this point we stress that the number of independent functions required for parametrizing the structure-dependent part is actually 18 instead of 21 as suggested in Ref. [3] (see Appendix A). The independent amplitudes B_i are functions of four invariants q^2 , q'^2 , $q \cdot q'$, and $q \cdot P$. The kinematics of the general VCS process with on-shell nucleons is completely specified by this set, and all other invariants can be expressed in terms of these variables.

So far we have considered both photons to be virtual. We will now discuss the amplitude $\mathcal{M}^{\gamma^* \gamma}$ of the VCS process $\gamma^* + N \rightarrow \gamma + N$, with real photons in the final state, i.e., $q'^2 = 0$

and $\epsilon' \cdot q' = 0$. In this specific case the tensors $T_3^{\mu\nu}$, $T_6^{\mu\nu}$, and $T_{19}^{\mu\nu}$ do not contribute to the amplitude. If we multiply the tensors $\tilde{T}_i^{\mu\nu} \equiv T_i^{\mu\nu}(q'^2 = 0)$ by the polarization vectors of both photons, we end up with 12 different structures which is the correct number of terms [5–7]. As a consequence, the invariant VCS matrix element $\mathcal{M}_B^{\gamma^*\gamma}$ can be written as

$$\mathcal{M}_B^{\gamma^*\gamma} = -ie^2 \bar{u}(p_f, S_f) \sum_{i=1}^{12} \epsilon_\mu \rho_i^{\mu\nu} \epsilon_\nu'^* f_i(q^2, q \cdot q', q \cdot P) u(p_i, S_i). \quad (7)$$

Equation (7), together with the explicit results for the quantities $\epsilon_\mu \rho_i^{\mu\nu} \epsilon_\nu'^*$ in Eq. (A10) of Appendix A, defines the general structure of the VCS amplitude with $q^2 \neq 0$ and $q'^2 = 0$. Since Eq. (7) is Lorentz invariant, it is frame independent, and it allows one to incorporate the constraints from the discrete symmetries in a rather simple way.

In the following we will work in the c.m. frame, i.e.,

$$\vec{p}_i = -\vec{q}, \quad \vec{p}_f = -\vec{q}', \quad (8)$$

and will use an orthonormal basis defined by the momenta of the photons,

$$\hat{e}_z = \hat{q}, \quad \hat{e}_y = \frac{\hat{q} \times \hat{q}'}{\sin \theta}, \quad \hat{e}_x = \hat{e}_y \times \hat{e}_z, \quad (9)$$

with θ denoting the scattering angle between \hat{q} and \hat{q}' .

The matrix element can be decomposed into a transverse and a longitudinal part,

$$\mathcal{M}_B^{\gamma^*\gamma} = ie^2 \chi_f^\dagger \left(\vec{\epsilon}_T \cdot \vec{M}_T + \frac{q^2}{\omega^2} \epsilon_z M_z \right) \chi_i, \quad (10)$$

where current conservation has been used,

$$q_\mu \epsilon^\mu = 0, \quad q_\mu M_B^{\mu\nu} = 0, \quad (11)$$

at the leptonic and the hadronic vertices, respectively. Note that in the VCS process discussed in this paper the polarization vector of the initial photon is generated by the electromagnetic transition current of the electron, $\epsilon^\mu = e \bar{u}_{e'} \gamma^\mu u_e / q^2$. Current conservation allows one to perform the gauge transformation $\epsilon^\mu \rightarrow a^\mu = \epsilon^\mu + \zeta q^\mu$. Then the choice $\zeta = -\vec{\epsilon} \cdot \vec{q} / \omega^2$ leads to the polarization vector

$$a^\mu = \left(0, \vec{\epsilon}_T + \frac{q^2}{\omega^2} \epsilon_z \hat{q} \right) \quad (12)$$

and thereby results in the specific form of $\mathcal{M}_B^{\gamma^*\gamma}$ in Eq. (10).

For the following discussion it is useful to decompose the VCS matrix element in Pauli space. We choose the parametrization and the corresponding amplitudes defined in Ref. [16]. The transverse and longitudinal matrix elements can, respectively, be parametrized in terms of eight and four structures,

$$\begin{aligned} \vec{\epsilon}_T \cdot \vec{M}_T &= \vec{\epsilon}'^* \cdot \vec{\epsilon}_T A_1 + \vec{\epsilon}'^* \cdot \hat{q} \vec{\epsilon}_T \cdot \hat{q}' A_2 \\ &+ i \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \vec{\epsilon}_T) A_3 + i \vec{\sigma} \cdot (\hat{q}' \times \hat{q}) \vec{\epsilon}'^* \cdot \vec{\epsilon}_T A_4 \\ &+ i \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \hat{q}) \vec{\epsilon}_T \cdot \hat{q}' A_5 + i \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \hat{q}') \vec{\epsilon}_T \cdot \hat{q}' A_6 \\ &- i \vec{\sigma} \cdot (\vec{\epsilon}_T \times \hat{q}') \vec{\epsilon}'^* \cdot \hat{q} A_7 - i \vec{\sigma} \cdot (\vec{\epsilon}_T \times \hat{q}) \vec{\epsilon}'^* \cdot \hat{q} A_8, \end{aligned} \quad (13)$$

$$\begin{aligned} M_z &= \vec{\epsilon}'^* \cdot \hat{q} A_9 + i \vec{\sigma} \cdot (\hat{q}' \times \hat{q}) \vec{\epsilon}'^* \cdot \hat{q} A_{10} \\ &+ i \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \hat{q}) A_{11} + i \vec{\sigma} \cdot (\vec{\epsilon}'^* \times \hat{q}') A_{12}. \end{aligned} \quad (14)$$

III. GENERALIZED POLARIZABILITIES

We now apply the general result of Eq. (7), in order to determine the number of independent polarizabilities emerging from the leading-order term of a consistent expansion of the residual amplitude $\mathcal{M}_B^{\gamma*\gamma}$ in the energy ω' of the outgoing, real photon [6]. For completeness we include the results of Ref. [9] for the spin-independent polarizabilities in our presentation.

The definition of the generalized polarizabilities in VCS is based upon the multipole representation of $\mathcal{M}_B^{\gamma*\gamma}$ [6,24]. In Ref. [6] the multipoles $H^{(\rho'L',\rho L)S}(\omega',\bar{q})$ were introduced, where ρ (ρ') denotes the type of the initial (final) photon ($\rho = 0$: charge, C; $\rho = 1$: magnetic, M; $\rho = 2$: electric, E). The initial (final) orbital angular momentum is characterized by L (L'), and the quantum number S distinguishes between non-spin-flip ($S = 0$) and spin-flip ($S = 1$) transitions.

According to the low-energy theorem for VCS [6,7], which is an extension of the famous low-energy theorem for RCS derived by Low [25], and Gell-Mann and Goldberger [26], $\mathcal{M}_B^{\gamma*\gamma}$ is at least linear in the energy of the real photon. If one restricts oneself to the lowest-order term in ω' , only electric and magnetic dipole radiation of the outgoing photon contributes to the amplitude. In that case selection rules for parity and angular momentum allow for three scalar multipoles ($S = 0$) and seven vector multipoles ($S = 1$), leading to the same number of generalized polarizabilities (see Ref. [6] for more details concerning the definition of the generalized polarizabilities).

It turns out that multipoles containing an electric transition can be replaced by more appropriate definitions. In the case of the outgoing photon only the leading term in $\omega' = |\vec{q}'|$ is considered. Therefore, Siegert's theorem [27], together with the continuity equation, offers the possibility to express the electric transitions in terms of the charge transitions. In contrast to the final state kinematics, one is interested in considering an arbitrary three-momentum \vec{q} of the virtual photon in the initial state, which allows for investigating the momentum dependence of the polarizabilities. Accordingly, one has to be careful when replacing the electric multipoles in the initial state with charge multipoles, because the difference between electric and charge multipoles must not be neglected. This leads to so-called mixed multipoles $\hat{H}^{(\rho'L',L)S}$ [6], which are no longer characterized by a well-defined multipole type of the incoming photon.

Bearing these considerations in mind, the generalized polarizabilities can be defined through

$$P^{(\rho'L',\rho L)S}(\bar{q}^2) = \left[\frac{1}{\omega'^L \bar{q}^L} H^{(\rho'L',\rho L)S}(\omega',\bar{q}) \right]_{\omega'=0} \quad (\rho, \rho' = 0, 1), \quad (15a)$$

$$\hat{P}^{(\rho'L',L)S}(\bar{q}^2) = \left[\frac{1}{\omega'^L \bar{q}^{L+1}} \hat{H}^{(\rho'L',L)S}(\omega',\bar{q}) \right]_{\omega'=0} \quad (\rho' = 0, 1), \quad (15b)$$

as functions of \bar{q}^2 [6]. Contrary to multipoles containing an electric transition in the initial state, the multipoles in Eqs. (15a) and (15b) have a path-independent limit as $\bar{q}, \omega' \rightarrow 0$. In particular, in the ω' - \bar{q} -plane the limits along the RCS line ($\bar{q} = \omega'$) and along the VCS line ($\omega' = 0$) coincide. This behavior of the multipoles makes it possible to relate, at $\bar{q} = 0$, some of the corresponding generalized polarizabilities to the polarizabilities defined in RCS. An extended discussion on the low-energy behavior of the multipoles and of the generalized polarizabilities can be found in Ref. [6].

Two of the three scalar polarizabilities can be understood as generalizations of the well-known electric (α) and magnetic (β) polarizabilities in RCS,

$$\alpha(\bar{q}^2) = -\frac{e^2}{4\pi} \sqrt{\frac{3}{2}} P^{(01,01)0}(\bar{q}^2), \quad (16a)$$

$$\beta(\bar{q}^2) = -\frac{e^2}{4\pi} \sqrt{\frac{3}{8}} P^{(11,11)0}(\bar{q}^2). \quad (16b)$$

To apply Eqs. (16a) and (16b) in Gaussian units one has to replace the factor $e^2/4\pi$ by $\alpha_{QED} = e_{Gauss}^2$. This replacement ensures that the numerical numbers of α and β in the Heaviside-Lorentz system and in the Gauss system are the same. Note that by definition the generalized polarizabilities of Ref. [6] do not depend on the value of e^2 .

Since we perform an expansion in ω' , we will introduce two variables,

$$\omega_0 = \omega|_{\omega'=0} = M - E_i = M - \sqrt{M^2 + \bar{q}^2}, \quad (17a)$$

$$Q_0^2 = Q^2|_{\omega'=0} = -q^2|_{\omega'=0} = -2M\omega_0. \quad (17b)$$

Following Guichon *et al.* [6], the leading terms of the amplitudes A_i from Eqs. (13) and (14) read

$$A_1 = \omega' \sqrt{\frac{E_i}{M}} \left[-\sqrt{\frac{3}{2}} \omega_0 P^{(01,01)0}(\bar{q}^2) - \frac{3}{2} \bar{q}^2 \hat{P}^{(01,1)0}(\bar{q}^2) - \sqrt{\frac{3}{8}} \bar{q} \cos \theta P^{(11,11)0}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18a)$$

$$A_2 = \omega' \sqrt{\frac{E_i}{M}} \left[\sqrt{\frac{3}{8}} \bar{q} P^{(11,11)0}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18b)$$

$$A_3 = \omega' \sqrt{\frac{E_i}{M}} \frac{3}{4} \left[-2\omega_0 P^{(01,01)1}(\bar{q}^2) + \sqrt{2} \bar{q}^2 \left[P^{(01,12)1}(\bar{q}^2) - \sqrt{3} \hat{P}^{(01,1)1}(\bar{q}^2) \right] + \left(-\bar{q} P^{(11,11)1}(\bar{q}^2) + \sqrt{\frac{3}{2}} \omega_0 \bar{q} P^{(11,02)1}(\bar{q}^2) + \sqrt{\frac{5}{2}} \bar{q}^3 \hat{P}^{(11,2)1}(\bar{q}^2) \right) \cos \theta \right] + \mathcal{O}(\omega'^2), \quad (18c)$$

$$A_4 = \omega' \sqrt{\frac{E_i}{M}} \frac{3}{4} \left[-\bar{q} P^{(11,11)1}(\bar{q}^2) - \sqrt{\frac{3}{2}} \omega_0 \bar{q} P^{(11,02)1}(\bar{q}^2) - \sqrt{\frac{5}{2}} \bar{q}^3 \hat{P}^{(11,2)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18d)$$

$$A_5 = -A_4, \quad (18e)$$

$$A_6 = \mathcal{O}(\omega'^2), \quad (18f)$$

$$A_7 = \omega' \sqrt{\frac{E_i}{M}} \frac{3}{4} \left[\bar{q} P^{(11,11)1}(\bar{q}^2) - \sqrt{\frac{3}{2}} \omega_0 \bar{q} P^{(11,02)1}(\bar{q}^2) - \sqrt{\frac{5}{2}} \bar{q}^3 \hat{P}^{(11,2)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18g)$$

$$A_8 = \omega' \sqrt{\frac{E_i}{M}} \left[-\frac{3}{\sqrt{2}} \bar{q}^2 P^{(01,12)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18h)$$

$$A_9 = \omega' \sqrt{\frac{E_i}{M}} \left[-\omega_0 \sqrt{\frac{3}{2}} P^{(01,01)0}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18i)$$

$$A_{10} = \omega' \sqrt{\frac{E_i}{M}} \left[-\frac{3\sqrt{3}}{2\sqrt{2}} \omega_0 \bar{q} P^{(11,02)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18j)$$

$$A_{11} = \omega' \sqrt{\frac{E_i}{M}} \left[-\frac{3}{2} \omega_0 P^{(01,01)1}(\bar{q}^2) + \frac{3\sqrt{3}}{2\sqrt{2}} \omega_0 \bar{q} \cos \theta P^{(11,02)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2), \quad (18k)$$

$$A_{12} = \omega' \sqrt{\frac{E_i}{M}} \left[\frac{\sqrt{3}\omega_0}{2\bar{q}} \left[P^{(11,00)1}(\bar{q}^2) - \sqrt{2}\bar{q}^2 P^{(11,02)1}(\bar{q}^2) \right] \right] + \mathcal{O}(\omega'^2). \quad (18l)$$

In the derivation we made use of the transformation (B1) (see Appendix B) between the A_i and the amplitudes defined in Ref. [6]. We note that the relation between the matrix element T^{VCS} in Ref. [6] and $\mathcal{M}_B^{\gamma^* \gamma}$ is given by

$$\mathcal{M}_B^{\gamma^* \gamma} = -ie^2 T^{VCS} / 2M. \quad (19)$$

Another low-energy expansion of the amplitudes A_i is obtained, if the covariant result of Eq. (7) is evaluated in the c.m. frame. Restricting ourselves to terms linear in ω' , the expansion reads

$$A_1 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-\omega_0 f_1 - 2M\bar{q}^2 f_3 + 2\omega_0 f_{10} + \left(\bar{q} f_1 + 2M\omega_0 \bar{q} f_3 - 2\frac{\omega_0^2}{\bar{q}} f_{10} \right) \cos \theta \right] + \mathcal{O}(\omega'^2), \quad (20a)$$

$$A_2 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-\bar{q} f_1 - 2M\omega_0 \bar{q} f_3 + 2\frac{\omega_0^2}{\bar{q}} f_{10} \right] + \mathcal{O}(\omega'^2), \quad (20b)$$

$$A_3 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-M\omega_0 f_5 + M\omega_0^2 f_8 - 2\omega_0 f_{10} - M\omega_0^2 f_{12} + \left(\frac{M\omega_0^2}{\bar{q}} f_5 - M\omega_0 \bar{q} f_8 + 2\bar{q} f_{10} + \frac{M\omega_0^3}{\bar{q}} f_{12} \right) \cos \theta \right] + \mathcal{O}(\omega'^2), \quad (20c)$$

$$A_4 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-4\frac{M\omega_0}{\bar{q}} f_{10} \right] + \mathcal{O}(\omega'^2), \quad (20d)$$

$$A_5 = -A_4, \quad (20e)$$

$$A_6 = \mathcal{O}(\omega'^2), \quad (20f)$$

$$A_7 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-\frac{M\omega_0^2}{\bar{q}} f_5 + M\omega_0 \bar{q} f_8 - 2\bar{q} f_{10} - \frac{M\omega_0^3}{\bar{q}} f_{12} \right] + \mathcal{O}(\omega'^2), \quad (20g)$$

$$A_8 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-8M^2\omega_0 f_6 - M\omega_0 f_7 - M\omega_0^2 f_8 - 4M^2\omega_0 f_9 + 2\omega_0 f_{10} - 4M\omega_0 f_{11} + M\omega_0^2 f_{12} \right] + \mathcal{O}(\omega'^2), \quad (20h)$$

$$A_9 = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-\omega_0 f_1 + 2M\bar{q}^2 f_2 + 4M\omega_0^2 f_6 + 2M\omega_0^2 f_9 - 2M\omega_0^2 f_{12} \right] + \mathcal{O}(\omega'^2), \quad (20i)$$

$$A_{10} = \omega' \sqrt{\frac{E_i + M}{2M}} \left[2\omega_0 \bar{q} f_4 - \frac{\omega_0^3}{2\bar{q}} f_5 - \frac{\omega_0 \bar{q}}{2} f_7 - 2\frac{\omega_0^2}{\bar{q}} f_{10} - 2\omega_0 \bar{q} f_{11} - \frac{M\omega_0^3}{\bar{q}} f_{12} \right]$$

$$+\mathcal{O}(\omega'^2), \quad (20j)$$

$$A_{11} = \omega' \sqrt{\frac{E_i + M}{2M}} \left[2\bar{q}^2 f_4 - \frac{\omega_0^2}{2} f_5 - \frac{\omega_0^2}{2} f_7 - 2\omega_0 f_{10} - 2\omega_0^2 f_{11} - 2M\omega_0^2 f_{12} \right. \\ \left. + \left(-2\omega_0 \bar{q} f_4 + \frac{\omega_0^3}{2\bar{q}} f_5 + \frac{\omega_0 \bar{q}}{2} f_7 + 2\frac{\omega_0^2}{\bar{q}} f_{10} + 2\omega_0 \bar{q} f_{11} + \frac{M\omega_0^3}{\bar{q}} f_{12} \right) \cos \theta \right] \\ +\mathcal{O}(\omega'^2), \quad (20k)$$

$$A_{12} = \omega' \sqrt{\frac{E_i + M}{2M}} \left[-\frac{M\omega_0^2}{\bar{q}} f_5 + \left(M\omega_0 \bar{q} - 2\frac{M^2\omega_0^2}{\bar{q}} \right) f_{12} \right] + \mathcal{O}(\omega'^2). \quad (20l)$$

Because of the expansion in ω' , the functions in Eqs. (20) must have the arguments $f_i = f_i|_{\omega'=0} = f_i(-Q_0^2, 0, 0)$.

Until now we have not used the transformation properties of the functions f_i with respect to photon crossing and the combination of charge conjugation and nucleon crossing, which can be obtained from Eqs. (A6) and (A11). In particular, the behavior with respect to charge conjugation leads to the conclusion that f_3 , f_4 , f_8 , and f_{10} are odd functions of $q \cdot P$ (see Eq. (A6d)). Consequently, they are at least linear in ω' and, therefore, do not contribute to the leading-order terms in Eqs. (20a)–(20l). Hence we can omit these four functions and derive relations between the polarizabilities by comparing the amplitudes A_i of the two different low-energy expansions (18a)–(18l) and (20a)–(20l) of $\mathcal{M}_B^{\gamma^* \gamma}$. To be specific, A_4 in Eq. (20d) vanishes to lowest order in ω' , thus relating the polarizabilities $P^{(11,11)1}$, $P^{(11,02)1}$, and $\hat{P}^{(11,2)1}$. Two further relations arise because the terms with and without $\cos \theta$ in the amplitudes A_1 and A_3 are, respectively, given by the same linear combinations of the f_i . An inspection of A_{11} and A_{12} yields a fourth relation: While Eqs. (20k) and (20l) contain only two independent linear combinations of the f_i , A_{11} and A_{12} in Eqs. (18k) and (18l) depend on three polarizabilities. Note that the identity $\omega_0^2 = \bar{q}^2 + 2M\omega_0$ enters into the derivation of the last relation. Altogether, we obtain four relations between the ten original generalized polarizabilities,

$$0 = \sqrt{\frac{3}{2}} P^{(01,01)0}(\bar{q}^2) + \sqrt{\frac{3}{8}} P^{(11,11)0}(\bar{q}^2) + \frac{3\bar{q}^2}{2\omega_0} \hat{P}^{(01,1)0}(\bar{q}^2), \quad (21a)$$

$$0 = P^{(11,11)1}(\bar{q}^2) + \sqrt{\frac{3}{2}} \omega_0 P^{(11,02)1}(\bar{q}^2) + \sqrt{\frac{5}{2}} \bar{q}^2 \hat{P}^{(11,2)1}(\bar{q}^2), \quad (21b)$$

$$0 = 2\omega_0 P^{(01,01)1}(\bar{q}^2) + 2\frac{\bar{q}^2}{\omega_0} P^{(11,11)1}(\bar{q}^2) - \sqrt{2} \bar{q}^2 P^{(01,12)1}(\bar{q}^2) + \sqrt{6} \bar{q}^2 \hat{P}^{(01,1)1}(\bar{q}^2), \quad (21c)$$

$$0 = 3\frac{\bar{q}^2}{\omega_0} P^{(01,01)1}(\bar{q}^2) - \sqrt{3} P^{(11,00)1}(\bar{q}^2) - \sqrt{\frac{3}{2}} \bar{q}^2 P^{(11,02)1}(\bar{q}^2). \quad (21d)$$

As is evident from the definition of the generalized polarizabilities in Eqs. (15a) and (15b) the relations (21a)–(21d) can only be applied along the VCS line $\omega' = 0$. The relation between the scalar polarizabilities in Eq. (21a) has already been derived in Ref. [9]. It is an important consequence of Eqs. (21a)–(21d) that six independent functions of \bar{q}^2 are sufficient to parameterize the structure-dependent VCS amplitude to lowest order in ω' .

We want to emphasize again that the four relations between the ten generalized polarizabilities are ultimately caused by charge conjugation in connection with nucleon crossing.

If we drop the assumption that this symmetry holds, the functions f_3 , f_4 , f_8 , and f_{10} give a contribution to the leading-order terms in Eqs. (20a)–(20l), and none of our four relations between the polarizabilities is valid any longer. In this sense we find complete agreement with the analysis of Ref. [6], because the constraint due to charge conjugation and nucleon crossing has not been taken care of in that reference.

At $\bar{q} = 0$, particular relations between the polarizabilities and their derivatives can be found by expanding Eqs. (21a)–(21d). We only discuss the most interesting cases: Three of the seven vector polarizabilities vanish at $\bar{q} = 0$,

$$P^{(01,01)1}(0) = P^{(11,11)1}(0) = P^{(11,00)1}(0) = 0. \quad (22)$$

These results follow, in part, from Eqs. (21b) and (21d), if one exploits the expansion $\omega_0 = -\bar{q}^2/2M + \mathcal{O}(\bar{q}^4)$. Equation (21d) only contains the information that a certain linear combination of $P^{(01,01)1}(0)$ and $P^{(11,00)1}(0)$ disappears. The fact that both polarizabilities vanish separately becomes obvious by comparing the angular-independent part of the amplitude A_{11} in Eqs. (18k) and (20k).

Combining Eqs. (21b) and (21c) enables us to eliminate $P^{(11,11)1}$. This leads to a relation between the remaining four vector polarizabilities,

$$P^{(01,12)1}(0) + \sqrt{3}P^{(11,02)1}(0) - \sqrt{3}\hat{P}^{(01,1)1}(0) - 2\sqrt{5}M\hat{P}^{(11,2)1}(0) = 0. \quad (23)$$

The relations between the generalized polarizabilities also imply that several multipoles are connected at small values of ω' . Making use of Eqs. (21a) and (21b) we list the two most striking examples,

$$\begin{aligned} H^{(21,21)0}(\omega', \bar{q}) &= \omega' \left[2\omega_0 P^{(01,01)0}(\bar{q}^2) + \sqrt{6}\bar{q}^2 \hat{P}^{(01,1)0}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2) \\ &= -\omega'\omega_0 P^{(11,11)0}(\bar{q}^2) + \mathcal{O}(\omega'^2) \\ &= -\frac{\omega_0}{\bar{q}} H^{(11,11)0}(\omega', \bar{q}) + \mathcal{O}(\omega'^2), \end{aligned} \quad (24a)$$

$$\begin{aligned} H^{(11,11)1}(\omega', \bar{q}) &= \omega'\bar{q} P^{(11,11)1}(\bar{q}^2) + \mathcal{O}(\omega'^2) \\ &= \omega' \left[-\sqrt{\frac{3}{2}}\omega_0\bar{q} P^{(11,02)1}(\bar{q}^2) - \sqrt{\frac{5}{2}}\bar{q}^3 \hat{P}^{(11,2)1}(\bar{q}^2) \right] + \mathcal{O}(\omega'^2) \\ &= H^{(11,22)1}(\omega', \bar{q}) + \mathcal{O}(\omega'^2). \end{aligned} \quad (24b)$$

These equations are based upon the low-energy expansion of the multipoles given in Ref. [6]. Obviously, charge conjugation leads, at least in VCS, to unexpected constraints between the multipoles, which go beyond the conditions due to parity and angular momentum conservation. Whether these constraints are limited to the lowest order in ω' is beyond the scope of our present investigation. An answer to this question would require both a multipole analysis including angular momenta $L' \geq 2$, and an extension of Eqs. (20a)–(20l) to higher orders in ω' .

In Ref. [9] it has been argued that the relation between the scalar electric and magnetic multipole (Eq. (24a)) vanishes in the static limit $M \rightarrow \infty$, which is obvious from the definition of ω_0 . However, the second equation (24b) is not affected by this limit. Accordingly,

while Eq. (24a) may be interpreted as a recoil effect, the connection between $H^{(11,11)1}$ and $H^{(11,22)1}$ seems to indicate an intrinsic property of the target.

From a practical point of view, the results in Eqs. (21a)–(21d) are very appropriate to test predictions for the generalized polarizabilities of models incorporating the required symmetries. Moreover, they can serve as constraints for experimental analyses.

With the exception of the electric polarizability $\alpha(\vec{q}^2)$, the measurement of individual polarizabilities requires polarization experiments. In the unpolarized case it has been proposed [6] to extract four linear combinations of the polarizabilities by measuring the structure functions

$$P_{LL}(\vec{q}) = -2\sqrt{6}MG_E(Q_0^2)P^{(01,01)0}(\vec{q}^2), \quad (25a)$$

$$P_{TT}(\vec{q}) = \frac{3}{2}G_M(Q_0^2)\left[2\omega_0P^{(01,01)1}(\vec{q}^2) + \sqrt{2}\vec{q}^2\left(P^{(01,12)1}(\vec{q}^2) + \sqrt{3}\hat{P}^{(01,1)1}(\vec{q}^2)\right)\right], \quad (25b)$$

$$P_{LT}(\vec{q}) = \sqrt{\frac{3}{2}}\frac{M\vec{q}}{\sqrt{Q_0^2}}G_E(Q_0^2)P^{(11,11)0}(\vec{q}^2) + \frac{\sqrt{3}\sqrt{Q_0^2}}{2\vec{q}}G_M(Q_0^2)\left(P^{(11,00)1}(\vec{q}^2) + \frac{\vec{q}^2}{\sqrt{2}}P^{(11,02)1}(\vec{q}^2)\right), \quad (25c)$$

$$P'_{LT}(\vec{q}) = \sqrt{\frac{3}{2}}\frac{M}{\sqrt{Q_0^2}}G_E(Q_0^2)\left(2\omega_0P^{(01,01)0}(\vec{q}^2) + \sqrt{6}\vec{q}^2\hat{P}^{(01,1)0}(\vec{q}^2)\right) - \frac{3}{2}\sqrt{Q_0^2}G_M(Q_0^2)P^{(01,01)1}(\vec{q}^2), \quad (25d)$$

with G_E and G_M denoting the electric and magnetic Sachs form factors, respectively. These structure functions describe, to lowest order in ω' , the interference between the non-Born and the Born plus Bethe-Heitler amplitude. By use of Eqs. (21a) and (21d) the structure functions P_{LT} and P'_{LT} turn out to be mutually dependent via the relation

$$P_{LT}(\vec{q}) + \frac{\vec{q}}{\omega_0}P'_{LT}(\vec{q}) = 0. \quad (26)$$

This indicates that in an unpolarized experiment there are only three independent structure functions containing five generalized polarizabilities.

IV. SUMMARY

We analyzed VCS off the nucleon in a covariant, model-independent formalism, which allowed us to include constraints from discrete symmetries in a natural way. We restricted our investigation to the so-called structure-dependent part which is obtained from the full amplitude by subtracting a separately gauge-invariant Born part involving the vertex of Eq. (4). We demonstrated that it is possible to parametrize the VCS invariant matrix element in such a fashion that the tensor structures as well as the corresponding amplitudes are free of kinematical singularities. Consequently, the amplitudes only contain information on the dynamics of the process to be explored by the experiment. We then focused on Compton scattering with a virtual, spacelike photon in the initial and a real photon in the

final state, because this process will be investigated in future experiments. Applying our covariant approach to particular kinematical scenarios we critically reviewed the formalism presently used in the analysis of VCS experiments below pion threshold [6]. We found that charge-conjugation symmetry in connection with nucleon crossing generates four relations among the ten originally proposed generalized polarizabilities of the nucleon. We further derived relations between the generalized polarizabilities at particular kinematical points. We hope that our results will facilitate future theoretical and experimental analysis. These results have already been quite valuable for the analysis of VCS within the framework of the linear sigma model [14,15] and HBChPT [16,17]. All constraints on the generalized polarizabilities derived in this paper were confirmed on the level of model calculations with these two effective Lagrangians, because they incorporate the relevant symmetries, gauge invariance and Lorentz invariance as well as the discrete symmetries. We consider this as an important check for both the model calculations and our general results.

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APPENDIX A: GENERAL FORM OF THE COMPTON TENSOR

A construction of the Compton tensor $M^{\mu\nu}$ of the most general VCS reaction $\gamma^* + N \rightarrow \gamma^* + N$ has been given by Tarrach [3]. Here, we sketch the main features of his derivation and extend it with respect to our considerations. The list of all possible tensor structures $K_i^{\mu\nu}$ of the most general Compton tensor is built up from the four independent Lorentz vectors q^μ , q'^μ , P^μ , γ^μ . Each structure $K_i^{\mu\nu}$ must be even with respect to parity transformations, because we consider only parity-conserving interactions. Furthermore, it is useful to choose the $K_i^{\mu\nu}$ with a well-defined behavior under photon crossing ($q \leftrightarrow -q'$, $\mu \leftrightarrow \nu$) and under the combination of nucleon crossing and charge conjugation C . With these assumptions one obtains 34 $K_i^{\mu\nu}$ (see Eq. (8) of [3] for the complete list)

$$K_1^{\mu\nu} = g^{\mu\nu}, \quad \dots, \quad K_{34}^{\mu\nu} = (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) Q \cdot \gamma + Q \cdot \gamma (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad (\text{A1})$$

where the structures $K_1^{\mu\nu} - K_{10}^{\mu\nu}$ would also appear in the derivation for a spin-0 particle. Note our definition of P and Q of Eq. (1) and the reversed order of μ and ν as compared with Ref. [3]. Using four-momentum conservation and Dirac's equation it is possible to express each other tensor in terms of the $K_i^{\mu\nu}$. Moreover, two nontrivial relations between several of the $K_i^{\mu\nu}$ hold [3], reducing the number of independent tensors to 32. Even if there is some freedom in the choice of the independent tensors, it is convenient to eliminate $K_{13}^{\mu\nu}$ and $K_{28}^{\mu\nu}$ [3], which will not appear in the following derivation any more. Counting the helicities of the four particles involved in the reaction one ends up with the same number $32 = (2 \times 2 \times 4 \times 4)/2$, where the division by 2 is due to parity conservation in boson-fermion scattering [28]. Since each photon is considered off shell, it has components of spin 1 and spin 0 and thus enters with four degrees of freedom into the counting [28].

In order to incorporate current conservation at both photon vertices [see Eq. (5)], one derives linear combinations of the $K_i^{\mu\nu}$, which then form the basis vectors of $M^{\mu\nu}$. For the spin-independent amplitude this procedure has been explained in more detail in Refs. [8] and [9]. In the construction of such gauge-invariant linear combinations it usually happens that poles in the independent invariants q^2 , q'^2 , $q \cdot q'$, and $q \cdot P$ of the VCS reaction arise, leading to unphysical zeros or constraints in the corresponding amplitudes of the basis vectors. A general solution developed by Bardeen and Tung [1] avoids this problem, which one encounters in different physical reactions. The application of this method to VCS results in 18 gauge-invariant and pole-free tensors [3],

$$T_1^{\mu\nu} = -q \cdot q' K_1^{\mu\nu} + K_3^{\mu\nu}, \dots, \quad (\text{A2a})$$

$$T_{18}^{\mu\nu} = K_{17}^{\mu\nu} - 2q \cdot P K_{25}^{\mu\nu} + \frac{q \cdot q'}{2} K_{34}^{\mu\nu}. \quad (\text{A2b})$$

The spin-independent tensors $T_1^{\mu\nu}, \dots, T_5^{\mu\nu}$ are the same as in Eq. (5) of Ref. [9], whereas the basis elements $T_6^{\mu\nu}, \dots, T_{18}^{\mu\nu}$ correspond to the tensors $\tau_6^{\mu\nu}, \dots, \tau_{18}^{\mu\nu}$ in Eq. (12) of Ref. [3], rewritten for our choice of P and Q in Eq. (1). Note that the number of these tensors also results from counting one longitudinal and two transverse degrees of polarization of the virtual photons, $18 = (2 \times 2 \times 3 \times 3)/2$.

The above considerations determine the general form of $M^{\mu\nu}$. In particular, the gauge invariant residual part $M_B^{\mu\nu}$ [see Eq. (3)] of the Compton tensor can be expressed in terms of the basis vectors in (A2a) and (A2b) according to

$$M_B^{\mu\nu} = \sum_{i=1}^{18} B'_i(q^2, q'^2, q \cdot q', q \cdot P) T_i^{\mu\nu}. \quad (\text{A3})$$

However, the above basis has one drawback. Though the tensors $T_i^{\mu\nu}$ are free of poles, the corresponding amplitudes B'_i still contain kinematical constraints. Such a basis is called “nonminimal” [3]. The nonminimality is due to the fact that it is impossible to make a transformation into an equivalent, pole-free basis without introducing any kinematical pole in the transformation matrix [3]. As a consequence, three further gauge-invariant and pole-free tensors exist, which can be obtained from $T_1^{\mu\nu}, \dots, T_{18}^{\mu\nu}$ only with factors carrying a single pole in $q \cdot q'$:

$$\begin{aligned} T_{19}^{\mu\nu} &= \frac{1}{q \cdot q'} \left[-q^2 q'^2 T_2^{\mu\nu} + (q \cdot P)^2 T_3^{\mu\nu} - q \cdot P \frac{q^2 + q'^2}{2} T_4^{\mu\nu} + q \cdot P \frac{q^2 - q'^2}{2} T_5^{\mu\nu} \right] \\ &= (q \cdot P)^2 K_2^{\mu\nu} + q^2 q'^2 K_6^{\mu\nu} - q \cdot P \frac{q^2 + q'^2}{2} K_9^{\mu\nu} - q \cdot P \frac{q^2 - q'^2}{2} K_{10}^{\mu\nu}, \end{aligned} \quad (\text{A4a})$$

$$\begin{aligned} T_{20}^{\mu\nu} &= \frac{1}{4q \cdot q'} \left[(q^2 - q'^2) T_{10}^{\mu\nu} - 2(q^2 + q'^2) T_{14}^{\mu\nu} + 2q \cdot P T_{15}^{\mu\nu} \right] \\ &= -\frac{q^2 - q'^2}{2} K_6^{\mu\nu} - \frac{q \cdot P}{2} K_{10}^{\mu\nu} + M \frac{q^2 - q'^2}{2} K_{21}^{\mu\nu} + M \frac{q^2 + q'^2}{2} K_{22}^{\mu\nu} - M q \cdot P K_{24}^{\mu\nu} \\ &\quad + \frac{q^2 + q'^2}{8} K_{27}^{\mu\nu} - \frac{q \cdot P}{4} K_{29}^{\mu\nu} - q \cdot P \frac{q^2 - q'^2}{4} K_{33}^{\mu\nu} + M \frac{q^2 - q'^2}{8} K_{34}^{\mu\nu}, \end{aligned} \quad (\text{A4b})$$

$$T_{21}^{\mu\nu} = \frac{1}{4q \cdot q'} \left[(q^2 + q'^2) T_{10}^{\mu\nu} - 2(q^2 - q'^2) T_{14}^{\mu\nu} + 2q \cdot P T_{16}^{\mu\nu} \right]$$

$$\begin{aligned}
&= -\frac{q^2 + q'^2}{2} K_6^{\mu\nu} + \frac{q \cdot P}{2} K_9^{\mu\nu} + M \frac{q^2 + q'^2}{2} K_{21}^{\mu\nu} + M \frac{q^2 - q'^2}{2} K_{22}^{\mu\nu} - M q \cdot P K_{23}^{\mu\nu} \\
&\quad + \frac{q^2 - q'^2}{8} K_{27}^{\mu\nu} - \frac{q \cdot P}{4} K_{30}^{\mu\nu} - q \cdot P \frac{q^2 + q'^2}{4} K_{33}^{\mu\nu} + M \frac{q^2 + q'^2}{8} K_{34}^{\mu\nu}. \tag{A4c}
\end{aligned}$$

The nonminimality of the basis in Eq. (A2) is reflected by the fact that in the case $q \cdot q' = 0$ the set of tensors in Eq. (A2) does not form a tensor basis any more, because some elements of the original basis become linearly dependent [3]. Unfortunately, the two kinematical scenarios we investigate for the analysis of VCS at small final photon energy ω' both imply $q \cdot q' = 0$.

For this reason, when constructing the tensor basis for the residual part $M_B^{\mu\nu}$, we will have to start with a tensor basis different from the one of Eq. (A2).

It turns out that if we use $T_{19}^{\mu\nu}$ instead of $T_5^{\mu\nu}$, $T_{20}^{\mu\nu}$ instead of $T_{15}^{\mu\nu}$, and $T_{21}^{\mu\nu}$ instead of $T_{16}^{\mu\nu}$, we obtain a tensor basis which is free of poles and zeroes and, thus, can also be used in the case $q \cdot q' = 0$. However, this new basis is not minimal either, because poles in the invariant $q \cdot P$ can create linear dependences among the basis elements in the Born part of the Compton tensor. However, this is not the case for the residual part, which we are interested in in this paper. The residual part of the Compton tensor reads

$$M_B^{\mu\nu} = \sum_{i \in J} B_i(q^2, q'^2, q \cdot q', q \cdot P) T_i^{\mu\nu}, \quad J = \{1, \dots, 21\} \setminus \{5, 15, 16\}. \tag{A5}$$

The corresponding amplitudes $B_i(q^2, q'^2, q \cdot q', q \cdot P)$ are free of kinematical constraints, in particular free of poles. This can be proved by means of considering their symmetry properties: The tensor $M_B^{\mu\nu}$ is invariant under photon crossing and the combination of charge conjugation with nucleon crossing [3]. Since the $T_i^{\mu\nu}$ exhibit definite transformation properties with respect to photon crossing and charge conjugation combined with nucleon crossing, the amplitudes B_i do as well. By means of the identities

$$B_i(q^2, q'^2, q \cdot q', q \cdot P) = +B_i(q'^2, q^2, q \cdot q', -q \cdot P) \tag{A6a}$$

$$(i = 1, 2, 3, 5, 8, 10, 13, 15, 18, 19, 21),$$

$$B_i(q^2, q'^2, q \cdot q', q \cdot P) = -B_i(q'^2, q^2, q \cdot q', -q \cdot P) \tag{A6b}$$

$$(i = 4, 6, 7, 9, 11, 12, 14, 16, 17, 20),$$

$$B_i(q^2, q'^2, q \cdot q', q \cdot P) = +B_i(q^2, q'^2, q \cdot q', -q \cdot P) \tag{A6c}$$

$$(i = 1, 2, 3, 8, 9, 10, 11, 14, 18, 19, 20, 21),$$

$$B_i(q^2, q'^2, q \cdot q', q \cdot P) = -B_i(q^2, q'^2, q \cdot q', -q \cdot P) \tag{A6d}$$

$$(i = 4, 5, 6, 7, 12, 13, 15, 16, 17),$$

the functions B_i can be divided into four classes, where in Eqs. (A6a) and (A6b) use has been made of the identity $q \cdot P = q' \cdot P$. We emphasize that Eqs. (A6c) and (A6d), which are crucial for the derivation of the relations between the generalized polarizabilities in Sec. III, may alternatively be derived by means of time reversal together with photon crossing [3].

For the definition of low-energy constants we need a general expansion of the B_i up to the order $\mathcal{O}(k^3)$ ($k \in \{q, q'\}$), which immediately follows from the transformation properties of Eqs. (A6a)–(A6d):

$$B_i = b_{i,0} + b_{i,2a}q \cdot q' + b_{i,2b}(q^2 + q'^2) + b_{i,2c}(q \cdot P)^2 + \mathcal{O}(k^4) \\ (i = 1, 2, 3, 8, 10, 18, 19, 21), \quad (\text{A7a})$$

$$B_i = b_{i,3}(q^2 - q'^2)q \cdot P + \mathcal{O}(k^4) \quad (i = 5, 13, 15), \quad (\text{A7b})$$

$$B_i = b_{i,2}(q^2 - q'^2) + \mathcal{O}(k^4) \quad (i = 9, 11, 14, 20), \quad (\text{A7c})$$

$$B_i = b_{i,1}q \cdot P + b_{i,3a}q \cdot Pq \cdot q' + b_{i,3b}q \cdot P(q^2 + q'^2) + b_{i,3c}(q \cdot P)^3 + \mathcal{O}(k^4) \\ (i = 4, 6, 7, 12, 16, 17). \quad (\text{A7d})$$

Such an expansion of the amplitudes in terms of the four-momenta of the photons has already been performed in Ref. [8] in connection with VCS from the pion.

From the above Taylor expansion, the fact that in the original representation

$$M_B^{\mu\nu} = \sum_{r \in R} C_r(q^2, q'^2, q \cdot q', q \cdot P) K_r^{\mu\nu}, \quad R = \{1, \dots, 34\} \setminus \{13, 28\}, \quad (\text{A8})$$

the functions C_r by definition are free of poles in the kinematical variables, and the symmetry properties of the C_r and $K_r^{\mu\nu}$ it follows that the functions $B_i(q^2, q'^2, q \cdot q', q \cdot P)$, $i \in J$, are free of poles. Furthermore, it can be shown that gauge invariance does not generate any additional kinematical constraints on these functions. Thus, Eq. (A5) contains a representation for $M_B^{\mu\nu}$ which satisfies all requirements — not only for our particular case $q \cdot q' = 0$, but for any choice of kinematical variables in $\gamma^* N \rightarrow \gamma^* N$. In particular, it is not necessary to use 3 additional functions as in [3]. Reexpressing this parametrization in the form of Eq. (A3), the functions B'_i read

$$B'_i = B_i \quad \text{for } i \in \{1, 6, 7, 8, 9, 11, 12, 13, 17, 18\}, \quad (\text{A9a})$$

$$B'_2 = B_2 - \frac{q^2 q'^2}{q \cdot q'} B_{19}, \quad (\text{A9b})$$

$$B'_3 = B_3 + \frac{(q \cdot P)^2}{q \cdot q'} B_{19}, \quad (\text{A9c})$$

$$B'_4 = B_4 - q \cdot P \frac{q^2 + q'^2}{2q \cdot q'} B_{19}, \quad (\text{A9d})$$

$$B'_5 = q \cdot P \frac{q^2 - q'^2}{2q \cdot q'} B_{19}, \quad (\text{A9e})$$

$$B'_{10} = B_{10} + \frac{q^2 - q'^2}{4q \cdot q'} B_{20} + \frac{q^2 + q'^2}{4q \cdot q'} B_{21}, \quad (\text{A9f})$$

$$B'_{14} = B_{14} - \frac{q^2 + q'^2}{2q \cdot q'} B_{20} - \frac{q^2 - q'^2}{2q \cdot q'} B_{21}, \quad (\text{A9g})$$

$$B'_{15} = \frac{q \cdot P}{2q \cdot q'} B_{20}, \quad (\text{A9h})$$

$$B'_{16} = \frac{q \cdot P}{2q \cdot q'} B_{21}. \quad (\text{A9i})$$

These equations follow from the definitions of $T_{19}^{\mu\nu}$, $T_{20}^{\mu\nu}$, and $T_{21}^{\mu\nu}$ in Eqs. (A4a)–(A4c).² We stress that the tensors $T_1^{\mu\nu}, \dots, T_{18}^{\mu\nu}$ still form a basis of the Compton tensor according

²Note that B_{19} is equivalent to the function B_6 in Ref. [9].

to Eq. (A3). The nonminimality of this basis is expressed in a specific kinematical behavior of the amplitudes B'_i , namely, some amplitudes contain poles in $q \cdot q'$. However, $M_B^{\mu\nu}$ is free of poles, despite the behavior of the B'_i . This is due to the fact that both the amplitudes B_i and the tensors $T_i^{\mu\nu}$, $i \in J$, do not carry any pole in the relativistic invariants.

For the discussion of $\mathcal{M}^{\gamma^*\gamma}$ we change the numbering by introducing tensors $\rho_i^{\mu\nu}$ in the following way:

$$\begin{aligned}\varepsilon_\mu \rho_1^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_1^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \varepsilon \cdot q' \varepsilon^{I*} \cdot q - q \cdot q' \varepsilon \cdot \varepsilon^{I*},\end{aligned}\tag{A10a}$$

$$\begin{aligned}\varepsilon_\mu \rho_2^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_2^{\mu\nu} \varepsilon_\nu^{I*} \\ &= q \cdot P(\varepsilon \cdot P \varepsilon^{I*} \cdot q + \varepsilon^{I*} \cdot P \varepsilon \cdot q') - q \cdot q' \varepsilon \cdot P \varepsilon^{I*} \cdot P - (q \cdot P)^2 \varepsilon \cdot \varepsilon^{I*},\end{aligned}\tag{A10b}$$

$$\begin{aligned}\varepsilon_\mu \rho_3^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_4^{\mu\nu} \varepsilon_\nu^{I*} \\ &= q \cdot P q^2 \varepsilon \cdot \varepsilon^{I*} - q \cdot P \varepsilon \cdot q \varepsilon^{I*} \cdot q - q^2 \varepsilon^{I*} \cdot P \varepsilon \cdot q' + q \cdot q' \varepsilon^{I*} \cdot P \varepsilon \cdot q,\end{aligned}\tag{A10c}$$

$$\begin{aligned}\varepsilon_\mu \rho_4^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_7^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \varepsilon \cdot P \varepsilon^{I*} \cdot P Q \cdot \gamma - q \cdot P(\varepsilon \cdot P \varepsilon^{I*} \cdot \gamma + \varepsilon^{I*} \cdot P \varepsilon \cdot \gamma) \\ &\quad + i q \cdot P \gamma_5 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\mu \varepsilon_\nu^{I*} Q_\alpha \gamma_\beta,\end{aligned}\tag{A10d}$$

$$\begin{aligned}\varepsilon_\mu \rho_5^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_8^{\mu\nu} \varepsilon_\nu^{I*} = -\varepsilon_\mu \tilde{T}_9^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \frac{1}{4} \varepsilon^{I*} \cdot P \varepsilon \cdot q Q \cdot \gamma + \frac{q^2}{4} (\varepsilon \cdot P \varepsilon^{I*} \cdot \gamma - \varepsilon^{I*} \cdot P \varepsilon \cdot \gamma) - \frac{q \cdot P}{2} \varepsilon \cdot q \varepsilon^{I*} \cdot \gamma \\ &\quad + \frac{i}{4} q^2 \gamma_5 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\mu \varepsilon_\nu^{I*} Q_\alpha \gamma_\beta,\end{aligned}\tag{A10e}$$

$$\begin{aligned}\varepsilon_\mu \rho_6^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_{10}^{\mu\nu} \varepsilon_\nu^{I*} \\ &= -2q \cdot q' \varepsilon \cdot P \varepsilon^{I*} \cdot P + q \cdot P(\varepsilon \cdot P \varepsilon^{I*} \cdot q + \varepsilon^{I*} \cdot P \varepsilon \cdot q') \\ &\quad + 2Mq \cdot q' (\varepsilon \cdot P \varepsilon^{I*} \cdot \gamma + \varepsilon^{I*} \cdot P \varepsilon \cdot \gamma) - 2Mq \cdot P(\varepsilon \cdot q' \varepsilon^{I*} \cdot \gamma + \varepsilon^{I*} \cdot q \varepsilon \cdot \gamma) \\ &\quad + i q \cdot P(\varepsilon \cdot q' \sigma^{\nu\alpha} \varepsilon_\nu^{I*} Q_\alpha - \varepsilon^{I*} \cdot q \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha) + 2i q \cdot q' q \cdot P \sigma^{\mu\nu} \varepsilon_\mu \varepsilon_\nu^{I*} \\ &\quad + 2i M q \cdot q' \gamma_5 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\mu \varepsilon_\nu^{I*} Q_\alpha \gamma_\beta,\end{aligned}\tag{A10f}$$

$$\begin{aligned}\varepsilon_\mu \rho_7^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_{11}^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \frac{1}{4} (\varepsilon \cdot P \varepsilon^{I*} \cdot q - \varepsilon^{I*} \cdot P \varepsilon \cdot q') Q \cdot \gamma - \frac{q \cdot q'}{2} (\varepsilon \cdot P \varepsilon^{I*} \cdot \gamma - \varepsilon^{I*} \cdot P \varepsilon \cdot \gamma) \\ &\quad + \frac{q \cdot P}{2} (\varepsilon \cdot q' \varepsilon^{I*} \cdot \gamma - \varepsilon^{I*} \cdot q \varepsilon \cdot \gamma),\end{aligned}\tag{A10g}$$

$$\begin{aligned}\varepsilon_\mu \rho_8^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_{12}^{\mu\nu} \varepsilon_\nu^{I*} = \varepsilon_\mu \tilde{T}_{13}^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \frac{q \cdot P}{2} \varepsilon \cdot q \varepsilon^{I*} \cdot q - \frac{q^2}{4} (\varepsilon \cdot P \varepsilon^{I*} \cdot q - \varepsilon^{I*} \cdot P \varepsilon \cdot q') - \frac{q \cdot q'}{2} \varepsilon^{I*} \cdot P \varepsilon \cdot q \\ &\quad - \frac{M}{2} \varepsilon \cdot q \varepsilon^{I*} \cdot q Q \cdot \gamma + M q \cdot q' \varepsilon \cdot q \varepsilon^{I*} \cdot \gamma - \frac{M}{2} q^2 (\varepsilon \cdot q' \varepsilon^{I*} \cdot \gamma - \varepsilon^{I*} \cdot q \varepsilon \cdot \gamma) \\ &\quad + \frac{i}{4} q^2 (\varepsilon \cdot q' \sigma^{\nu\alpha} \varepsilon_\nu^{I*} Q_\alpha - \varepsilon^{I*} \cdot q \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha) + \frac{i}{2} q \cdot q' q^2 \sigma^{\mu\nu} \varepsilon_\mu \varepsilon_\nu^{I*},\end{aligned}\tag{A10h}$$

$$\begin{aligned}\varepsilon_\mu \rho_9^{\mu\nu} \varepsilon_\nu^{I*} &= \varepsilon_\mu \tilde{T}_{14}^{\mu\nu} \varepsilon_\nu^{I*} \\ &= \frac{q \cdot P}{2} (\varepsilon \cdot P \varepsilon^{I*} \cdot q - \varepsilon^{I*} \cdot P \varepsilon \cdot q') - M q \cdot q' (\varepsilon \cdot P \varepsilon^{I*} \cdot \gamma - \varepsilon^{I*} \cdot P \varepsilon \cdot \gamma)\end{aligned}$$

$$\begin{aligned}
& +Mq \cdot P(\varepsilon \cdot q' \varepsilon'^* \cdot \gamma - \varepsilon'^* \cdot q \varepsilon \cdot \gamma) + \frac{i}{2} q \cdot q' (\varepsilon \cdot P \sigma^{\nu\alpha} \varepsilon'_\nu Q_\alpha + \varepsilon'^* \cdot P \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha) \\
& - \frac{i}{2} q \cdot P(\varepsilon \cdot q' \sigma^{\nu\alpha} \varepsilon'_\nu Q_\alpha + \varepsilon'^* \cdot q \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha), \tag{A10i}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_\mu \rho_{10}^{\mu\nu} \varepsilon'_\nu &= \varepsilon_\mu \tilde{T}_{17}^{\mu\nu} \varepsilon'^*_\nu \\
&= -2q \cdot P \varepsilon \cdot \varepsilon'^* + \varepsilon \cdot P \varepsilon'^* \cdot q + \varepsilon'^* \cdot P \varepsilon \cdot q' + 2M \varepsilon \cdot \varepsilon'^* Q \cdot \gamma \\
&\quad - 2M(\varepsilon \cdot q' \varepsilon'^* \cdot \gamma + \varepsilon'^* \cdot q \varepsilon \cdot \gamma) - i \varepsilon \cdot q' \sigma^{\nu\alpha} \varepsilon'_\nu Q_\alpha + i \varepsilon'^* \cdot q \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha \\
&\quad - 2iq \cdot q' \sigma^{\mu\nu} \varepsilon_\mu \varepsilon'^*_\nu, \tag{A10j}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_\mu \rho_{11}^{\mu\nu} \varepsilon'_\nu &= \varepsilon_\mu \tilde{T}_{18}^{\mu\nu} \varepsilon'^*_\nu \\
&= (\varepsilon \cdot P \varepsilon'^* \cdot q + \varepsilon'^* \cdot P \varepsilon \cdot q') Q \cdot \gamma - 2q \cdot P(\varepsilon \cdot q' \varepsilon'^* \cdot \gamma + \varepsilon'^* \cdot q \varepsilon \cdot \gamma) \\
&\quad + 2iq \cdot q' \gamma_5 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\mu \varepsilon'^*_\nu Q_\alpha \gamma_\beta, \tag{A10k}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_\mu \rho_{12}^{\mu\nu} \varepsilon'_\nu &= \varepsilon_\mu \tilde{T}_{20}^{\mu\nu} \varepsilon'^*_\nu = \varepsilon_\mu \tilde{T}_{21}^{\mu\nu} \varepsilon'^*_\nu \\
&= -\frac{q^2}{2} \varepsilon \cdot P \varepsilon'^* \cdot P + \frac{q \cdot P}{2} \varepsilon'^* \cdot P \varepsilon \cdot q + M q^2 \varepsilon \cdot P \varepsilon'^* \cdot \gamma - M q \cdot P \varepsilon \cdot q \varepsilon'^* \cdot \gamma \\
&\quad - \frac{i}{4} q^2 (\varepsilon \cdot P \sigma^{\nu\alpha} \varepsilon'_\nu Q_\alpha + \varepsilon'^* \cdot P \sigma^{\mu\alpha} \varepsilon_\mu Q_\alpha) + \frac{i}{2} q \cdot P \varepsilon \cdot q \sigma^{\nu\alpha} \varepsilon'_\nu Q_\alpha + \frac{i}{2} q^2 q \cdot P \sigma^{\mu\nu} \varepsilon_\mu \varepsilon'^*_\nu \\
&\quad + \frac{i}{2} M q^2 \gamma_5 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\mu \varepsilon'^*_\nu Q_\alpha \gamma_\beta. \tag{A10l}
\end{aligned}$$

The sign of the Levi-Civita symbol is fixed by $\varepsilon_{0123} = -\varepsilon^{0123} = 1$, and $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ is the usual abbreviation for the commutator of the Dirac matrices. Because of parity conservation Eqs. (A10a)–(A10l) do not contain any pseudoscalar structures.

In analogy with the tensors, one can replace the $B_i(q^2, 0, q \cdot q', q \cdot P)$ by 12 amplitudes $f_i = f_i(q^2, q \cdot q', q \cdot P)$,

$$\begin{aligned}
f_1 &= B_1, \quad f_2 = B_2, \quad f_3 = B_4, \quad f_4 = B_7, \quad f_5 = B_8 - B_9, \quad f_6 = B_{10}, \\
f_7 &= B_{11}, \quad f_8 = B_{12} + B_{13}, \quad f_9 = B_{14}, \quad f_{10} = B_{17}, \quad f_{11} = B_{18}, \quad f_{12} = B_{20} + B_{21}. \tag{A11}
\end{aligned}$$

APPENDIX B: AMPLITUDE SETS IN VIRTUAL COMPTON SCATTERING

Throughout this work we have applied the set of amplitudes defined in Eqs. (13) and (14). The relation to the convention of Ref. [6] is given by

$$\begin{aligned}
A_1 &= a^t, \\
A_2 &= a^{t'}, \\
A_3 &= -\sin \theta b_1^t + \sin \theta \cos \theta b_1^{t'} - \sin \theta b_2^{t'} - \sin^2 \theta b_3^{t'}, \\
A_4 &= -\frac{1}{\sin \theta} b_2^t, \\
A_5 &= \frac{1}{\sin \theta} (-\cos \theta b_1^t + b_1^{t'} - \cos \theta b_2^{t'}), \\
A_6 &= \frac{1}{\sin \theta} (b_1^t - \cos \theta b_1^{t'} + b_2^{t'}), \\
A_7 &= \frac{1}{\sin \theta} (-\cos \theta b_1^t + \cos^2 \theta b_1^{t'} - \cos \theta b_2^{t'} + \sin \theta b_3^t - \sin \theta \cos \theta b_3^{t'}),
\end{aligned}$$

$$\begin{aligned}
A_8 &= \frac{1}{\sin \theta} (\cos^2 \theta b_1^t - \cos \theta b_1^{t'} + b_2^{t'} - \sin \theta \cos \theta b_3^t + \sin \theta b_3^{t'}) , \\
A_9 &= a^l , \\
A_{10} &= \frac{1}{\sin \theta} (\cos \theta b_1^l - b_2^l - \sin \theta b_3^l) , \\
A_{11} &= \sin \theta b_1^l + \cos \theta b_3^l , \\
A_{12} &= -b_3^l .
\end{aligned} \tag{B1}$$

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